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On Isomorphic Classification of Spaces $s\widehat{\otimes}E'_\infty(a)$

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Abstract

Some new topological invariants, suggested earlier by Zahariuta, are used here for isomorphic classification of mixed F and DF tensor products of power series spaces. In particular some new results are obtained for vector-valued C^∞ spaces with values in a dual power series space $E'_\infty(a)$, which coincide up to isomorphism, with the space $L(E_\infty(a), s)$ of all linear continuous operators from $E_\infty(a)$ to s. Complete proofs will be published later.

Introduction

Let C_X^∞ denote the space of all infinitely differentiable functions defined on the interval $[-1, 1]$ with values in a given locally convex space X. Usually C_X^∞ is endowed with the topology of uniform convergence of functions on the interval $[-1, 1]$ with all their derivatives in every continous seminorm of X (cf.[9]). We have [9] the isomorphisms

$$C_X^\infty \simeq s\widehat{\otimes}X \simeq L(s', X).$$

Here and throughout s is the space of all rapidly decreasing sequences, $X\widehat{\otimes}Y$ the complete projective tensor product and $L(X, Y)$ the space of continuous linear operators from X into Y equipped with the topology of uniform convergence on bounded subsets of X. In particular, X' stands for $L(X, \mathbb{K})$, where $\mathbb{K}$ is the scalar field.

Our purpose will be to characterize the isomorphism $C_X^\infty \simeq C_Y^\infty$ in terms of the spaces X and Y. Valdivia has shown in [16] that if C_X^∞ is isomorphic to a complemented subspace of C_Y^∞ and C_Y^∞ in turn is isomorphic to a complemented subspace of X, then $C_X^\infty \simeq C_Y^\infty$. Using this result a simple application of the decomposition method of Aytuna, Krone and Terzioğlu [1] gives $C_X^\infty \simeq s$ whenever X is a complemented subspace of s.

In contrast, it was shown in [20], [21] that even in the simple case of a nuclear finite type power series space $X = E_0(a)$, the structure of C_X^∞ as a Frechét space depends on X

in a quite delicate way. This case will be treated in section 2. Those two diverse answers indicate that to study the general case, even if we restrict attention to the class C_X^∞, with X a nuclear Fréchet space, would not be very promising. Therefore we restrict our attention mainly to some natural classes such as C_X^∞, where X is the dual of a nuclear power series space $X \simeq E'_\infty(a)$. In the last case we have

$$C_X^\infty \simeq s\hat{\otimes}E'_\infty(a) \simeq L\left(E_\infty(a), s\right) \tag{0.1}$$

which gives us extra motivation to study this case. Related to this class, we also consider the class

$$s'\hat{\otimes}E_\infty(a) \simeq L\left(s, E_\infty(a)\right) \tag{0.2}$$

In a more general setting we consider the problem of isomorphic classification of the class of tensor products

$$E_\infty(a)\hat{\otimes}E'_\infty(b) \tag{0.3}$$

which covers both classes (0.1) and (0.2).

To classify the spaces (0.3) we introduce new linear topological invariants based on the idea suggested by Zahariuta in [19], [20] (see also [4], [5]). This may be roughly summarized as follows: starting from a given collection of absolutly convex bounded subsets, we construct by some invariant manner another collection of absolutely convex sets with many parameters. To this collection we apply the classical invariant characteristics (ϵ-entropy, n-diameters or their equivalents: see, for example [10], [14]). This approach yields the invariant characteristics for Köthe spaces, considered earlier in [17], [18] and initiated by Mitiagin's results [12], but in a form more convenient for our purpose.

Consideration of multiparameter invariant characteristics give us considerably more information about spaces than classical invariants (approximative, diametral dimensions or their variations based on one-parameter characteristics), did (see, for example, [2], [3], [6], [10], [11], [12], [15]).

Here we use as a basic one a collection of absolutely convex sets in $X\hat{\otimes}Y$, which corresponds (in our case) to some basis of equicontinuous sets in $L(Y^*, X)$. It is useful to compare this view with previous results [7] on necessary conditions of isomorphism of spaces $E_\alpha(a)\hat{\otimes}E'_\beta(b)$ which were based on more traditional conderations, dealing with neighborhoods of zero.

To construct an isomorphism or an isomorphic imbedding for a given pair of spaces $X = E_\infty(a)\hat{\otimes}E'_\infty(b)$ and $Y = E_\infty(\widetilde{a})\hat{\otimes}E'_\infty(\widetilde{b})$ we use here the method suggested in [18] but in considerably simplified form (in the spirit of revised English version of [20], which will appear in Turkish Math. J.).

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1. Preliminaries

1.1.

Let $A = (a_{i\lambda})_{i\in I,\lambda\in\Lambda}$ be a Köthe matrix, where I is a countable set (often $I = \mathbb{N}$). Λ is a directed set $a_{i\lambda} \geq 0$. We also have $a_{i\lambda} \leq a_{i\mu}$ if $\lambda \leq \mu$ and $\sup\{a_{i\lambda} : \lambda \in \Lambda\} > 0$, $i \in I$. By $K(A)$ we denote the Köthe space generated by A, i.e., the locally convex space of all sequences $x = (\xi_i)$ such that for every $\lambda \in \Lambda$

$$|x|_\lambda = \sum_{i\in I} |\xi_i|\, a_{i\lambda} < \infty \tag{1.1}$$

equipped with the seminorms (1.1). As usual, $e = (e_i)$ denotes the canonical basis of $K(A)$. In particular for $a = (a_i)$ by $E_0(a)$ and by $E_\infty(a)$ we denote the power series spaces of finite and infinite type, which, are Köthe spaces generated by the matrices $(\exp(-\frac{1}{p}a_i))$ and $(\exp(pa_i))$, $p \in \mathbb{N}$, respectively, see, for example [9].

If $A = (a_{i\lambda})_{i\in I,\lambda\in\Lambda}$, $B = (b_{j\mu})_{j\in J,\mu\in M}$ are two Köthe matrices, then the tensor product

$$K(A)\hat{\otimes}K(B) \simeq K(C),$$

where $C = (c_{(i,j),(\lambda,\mu)} = a_{i\lambda}b_{j\mu})$, with $(i,j) \in I \times J$ and $(\lambda,\mu) \in \Lambda \times M$. The above ismorphism is obtained by identifying the basis sequence $(e_i \otimes e_j)$ of $K(A)\hat{\otimes}K(B)$ with the natural basis (e_{ij}) of $K(C)$.

1.2.

A continuous linear operator $T\colon K(A) \to K(B)$ is said to be *quasidiagonal* if

$$T(e_i) = t_i e_{\sigma(i)}, \quad i \in I, \tag{1.2}$$

where $\sigma : I \to J$ and t_i is a scalar. In particular T is *diagonal* if $I = J$ and σ is an identity. If $\sigma\colon I \to J$ is a bijection and $t_i \equiv 1$, then T is said to be *permutative*. For Köthe spaces $X = K(A)$, $Y = K(B)$ we shall use the notation $X \overset{qd}{\simeq} Y$, $X \overset{d}{\simeq} Y$ and $X \overset{p}{\simeq} Y$ if there is an isomorphism $T\colon X \to Y$ which is respectively quasidiagonal, diagonal or permutative. We shall use the notation $X \hookrightarrow Y$ if there is an isomorphic imbedding $T\colon X \to Y$. If T is also quasidiagonal we write $X \overset{qd}{\simeq} Y$. In this context we need the following fact which was stated in [18] but was considered earlier in [12] in an implicit form.

Proposition 1.1. *If $K(A) \overset{qd}{\hookrightarrow} K(B)$ and $K(B) \overset{qd}{\hookrightarrow} K(A)$, then we have* $K(A) \overset{qd}{\simeq} K(B)$

1.3.

We shall identify the inductive limit

$$E'_\infty(a) = \lim \operatorname{ind} l^1(\exp -pa_i)$$

with the Köthe space $K(A)$, $A = (a_{i\pi})$, where

$$a_{i\pi} = \exp(-\pi_i a_i)$$

and $\pi = (\pi_i)$ runs through the directed set

$$\Pi^\infty = \{\pi = (\pi_i) : \lim \pi_i = \infty\}.$$

Π^∞ has the natural order $\lambda \le \mu$ if $\mu_i \le \lambda_i$ for all $i \in I$. In case $E_\infty(a)$ is nuclear, $E^*_\infty(a)$ can be naturally identified with $E'_\infty(a)$ ([7]).

1.4.

For a given sequence $a = (a_i)$, $a_i \ge 1$, we consider the following counting functions:

$$m_a(\tau, t) = |\{i : \tau < a_i \le t\}| \tag{1.3}$$

$$m_a(t) = |\{i : a_i \le t\}| \tag{1.4}$$

where $|A|$ denotes the cardinality of the set A, if A is finite, and equals to ∞ otherwise.We shall also use the following characteristics of lacunarity:

$$n_a(\tau, t) = \begin{cases} 1 & \text{if } m_a(\tau, t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

We shall write $m_a \approx m_b$ or $n_a \approx n_b$ if there exists a constant $c > 0$ such that

$$m_a(t) \le m_b(ct), \ m_a(t) \le m_b(ct), \ t \ge 1 \tag{1.5}$$

or, respectively,

$$n_a(\tau, t) \le n_b(\frac{\tau}{c}, ct), \ n_b(\tau, t) \le n_a(\frac{\tau}{c}, ct), \qquad 1 \le \tau < t < \infty \tag{1.6}$$

For non-decreasing sequence $a = (a_i)$ and $b = (b_i)$ the relation (1.5) is equivalent to the following condition:

$$\frac{b_i}{c} \leq a_i \leq cb_i, \ \ i \in N \tag{1.7}$$

with the same constant c. If for arbitary sequences a and b the relation (1.7) holds for some constant c, we say a and b are weakly equivalent and use the notation $a_i \asymp b_i$ or $a \asymp b$ in this case. If (1.6) holds, we say a and b have the *same lacunarities* or are identical in lacunarity.

The following simple result about the characteristic of lacunarity will be useful.

Proposition 1.2. *Let $a = (a_i)$, $b = (b_i)$ be sequences with $a_i \geq 1$, $b_i \geq 1$. The following statements are equivalent:*

(i) *There is a $\Delta > 0$ such that*

$$n_a(\tau, t) \leq n_b(\frac{\tau}{\Delta}, \Delta t), \ \ 1 \leq \tau < t < \infty \tag{1.8}$$

(ii) *For every $A > 1$ there is $B > 0$ with*

$$n_a(\frac{t}{A}, At) \leq n_b(\frac{t}{B}, Bt), \ \ t \geq 1$$

(iii) *There exists $A > 1$ and $B > 0$ with*

$$n_a(\frac{t}{A}, At) \leq n_b(\frac{t}{B}, Bt), \ \ t \geq 1$$

(iv) *There exists $A > 1$ and $B > 0$ with*

$$n_a(A^{2m-1}, A^{2m+1}) \leq n_b(B^{-1}A^{2m-1}, BA^{2m+1}), \ \ m \in \mathbb{Z}^n{}_+$$

2. Power Series Space-Valued Case

We will consider in detail the isomorphic classification of the spaces C_X^∞ when X is a power series space of infinite or finite type. First, we deal with the infinite type which is quite simple.

In fact we can consider C_X^∞ where X is a complemented subspace of s. It is not known if X has a basis, but if it does, then X is a nuclear power series space of infinite type.

Proposition 2.1. *If X is a complemented subspace of s, then $C_X^\infty \simeq s$.*

This fact follows from [1], since $C_X^\infty \simeq s\widehat{\otimes}X$ is a complemented subspace of s and its diametral dimension coincides with the diametral dimension of s.

In contrast to the above, the spaces C_X^∞, $X = E_0(a)$, have more intricate topological structure. Here the characteristic of lacunarity distinguishes isomorphic classes.

Proposition 2.2. *[20, 21] Let $E_0(a)$ be nuclear. Then*

$$s\widehat{\otimes}E_0(a) \simeq s\widehat{\otimes}E_0(b)$$

if and only if the following two conditions hold:

(1) $E_0(b)$ is nuclear,

(2) $n_a \approx n_b$, i.e. a and b are identical in lacunarity.

It is of interest to compare the preceeding result with the proposition in [13] on page 309.

3. Mixed F- and DF-Tensor Products

Here we state results, which will be proved in our more detailed paper [8].

Theorem 3.1. *Let $E_\infty(b)$ and $E_\infty(\widetilde{b})$ be two nuclear power series spaces where b and $\widetilde{b}$ are non-decreasing. Then $s\widehat{\otimes}E'_\infty(b) \simeq s\widehat{\otimes}E'_\infty(\widetilde{b})$ if and only if the sequences b and $\widetilde{b}$ are identical in lacunarity.*

We say that a sequence $b = (b_i)$ is non-lacunary if b and (i) are identical in lacunarity. For a non-decreasing sequence $b = (b_i)$ this is equivalent to

$$\limsup \frac{b_{i+1}}{b_i} < \infty$$

Hence the following statement is an immediate consequence of our theorem.

Corollary 3.2. *Let b be as in Theorem 3.1. Then*

$$s\widehat{\otimes}E'_\infty(b) \simeq s \otimes s'$$

if and only if b is non-lacunary.

The classification of spaces $s'\widehat{\otimes}E_\infty(a)$ depends on a in a more intricate fashion as in the case we have discussed. However, when the non-decreasing positive sequences $a = (a_i)$, $\widetilde{a} = (\widetilde{a}_i)$ satisfy the following stronger condition:

$$\ln i = o(a_i),\ \ \ln i = o(\widetilde{a}_i) \tag{3.1}$$

we have an analogue of Theorem 3.1.

Theorem 3.3. *Let a and $\widetilde{a}$ satisfy (3.1). Then $X = s'\widehat{\otimes}E_\infty(a)$ is isomorphic to $Y = s'\widehat{\otimes}E_\infty(\widetilde{a})$ if and only if a and $\widetilde{a}$ have the same lacunarity.*

For $c = (i)$, the space $E_\infty(c)$ is isomorphic to the space of entire functions $\mathcal{O}(\mathbb{C})$, c satisfies the condition (3.1) and is non-lacunary. Therefore we have the following fact.

Corollary 3.4. *Let a satisfy (3.1) and be non-lacunary. Then we have*

$$s'\widehat{\otimes}E_\infty(a) \simeq s'\widehat{\otimes}E_\infty(c) \simeq s' \otimes \mathcal{O}(\mathbb{C}).$$

Although $(\ln i)$ is non-lacunary, obviously it does not satisfy (3.1). In fact $s'\widehat{\otimes}s$ has an exceptional position in the class of spaces $s'\widehat{\otimes}E_\infty(a)$.

Theorem 3.5. *For the isomorphism $s' \otimes s \simeq s' \otimes E_\infty(a)$, it is necessary and sufficient that $s \simeq E_\infty(a)$.*

Corollary 3.6. *We have*

$$L(s, s) \simeq L(s, E_\infty(a))$$

if and only if $s \simeq E_\infty(a)$.

The preceeding results are derived from the following more general and rather technical result dealing with the isomorphism of spaces $E_\infty(a)\widehat{\otimes}E'_\infty(b)$.

Theorem 3.7. *Let $X = E_\infty(a)\widehat{\otimes}E'_\infty(b)$, $Y = E_\infty(\widetilde{a})\widehat{\otimes}E'_\infty(\widetilde{b})$ and $T : Y \to X$ be an isomorphism. Then $\exists\,\Delta\,\forall\,\widetilde{\varepsilon}\,\exists\,\varepsilon\,\forall\,\delta\,\exists\,\widetilde{\delta}$ such that the following inequalities (3.2)–(3.5) are true:*

$$\left|\left\{(i,j) : \delta \le \frac{b_j}{a_i + b_j} \le \varepsilon,\ \tau \le a_i + b_j \le t\right\}\right| \le$$
$$\left|\left\{(k,l) : \widetilde{\delta} \le \frac{\widetilde{b}_l}{\widetilde{a}_k + \widetilde{b}_l} \le \widetilde{\varepsilon},\ \frac{\tau}{\Delta} \le \widetilde{a}_k + \widetilde{b}_l \le \Delta t\right\}\right|,\ \tau \ge \tau_0 \qquad (3.2)$$

$$\left|\left\{(i,j) : \delta \le \frac{b_j}{a_i + b_j},\ \tau \le a_i + b_j \le t\right\}\right| \le$$
$$\left|\left\{(k,l) : \widetilde{\delta} \le \frac{\widetilde{b}_l}{\widetilde{a}_k + \widetilde{b}_l},\ \frac{\tau}{\Delta} \le \widetilde{a}_k + \widetilde{b}_l \le \Delta t\right\}\right|,\ \tau \ge \tau_0 \qquad (3.3)$$

$$\left|\left\{(i,j): \frac{b_j}{a_i+b_j} \le \varepsilon,\ \tau \le a_i+b_j \le t\right\}\right| \le \left|\left\{(k,l): \frac{\widetilde{b}_l}{\widetilde{a}_k+\widetilde{b}_l} \le \widetilde{\varepsilon},\ \frac{\tau}{\Delta} \le \widetilde{a}_k+\widetilde{b}_l \le \Delta t\right\}\right| \qquad (3.4)$$

$$|\{(i,j): \tau \le a_i+b_j \le t\}| \le \left|\left\{(k,l): \frac{\tau}{\Delta} \le \widetilde{a}_k+\widetilde{b}_l \le \Delta t\right\}\right|$$

Some quantifiers before absent parameters need to be omitted; the constant τ_0 depends on all participating parameters.

Remark. If t depends on $\tau : t = \varphi(\tau)$, $\tau \ge 1$, then the restriction $\tau \ge \tau_0$ can be removed everywhere in Theorem 3.7. Indeed, let, for example, the relation (3.3) hold with $t = \varphi(\tau)$ and $\tau \ge \tau_0$. Then we choose, instead of $\widetilde{\delta}$ some smaller constant $\widetilde{\delta}' > 0$, such that $\widetilde{\delta}'\ \Delta\ \varphi(\tau_0) \le 1$ and get the inequality (3.3) with $\widetilde{\delta}'$ instead of $\widetilde{\delta}$ without any restriction on τ.

Last theorem has the following partial converse.

Theorem 3.8. *Let X, Y be as in Theorem 3.7 and assume the conditions (3.2), (3.3) and (3.4) are valid. Then $X \overset{qd}{\lesssim} Y^9$.*

With some restrictions on X, Y we get the following criteria of isomorphism.

Theorem 3.9. *Let X, Y be as in Theorem 3.7 and $X \overset{qd}{\simeq} X^2$, $Y \overset{qd}{\simeq} Y^2$. Then the following statements are equivalent:*

(i) $X \simeq Y$

(ii) $X \overset{(qd)}{\simeq} Y$

(iii) the inequalities (3.2),(3.3) are true together with the inequalities, which are obtained from these ones by interchanging a, b, (i,j) with $\widetilde{a}$, $\widetilde{b}$, (k,l) respectively

Proof. Because (ii) $\Rightarrow$ (i) is obvious and (i) $\Rightarrow$ (iii) follows from Theorem 3.7, we need only to prove (iii) $\Rightarrow$ (ii). Since $X \overset{qd}{\simeq} X^2$ implies $X \overset{qd}{\simeq} X^{10}$ we get with Theorem 3.8 that $Y \overset{qd}{\lesssim} X^{10} \overset{qd}{\lesssim} X$, i.e. $Y \overset{qd}{\lesssim} X$. By symmetry we get also $X \overset{qd}{\lesssim} Y$. Hence Proposition 1.1 implies $X \overset{qd}{\simeq} Y$. □

Corollary 3.10. *Let X, Y be as in Theorem 3.7 and assume one of the following additonal conditions holds:*

$$a_{2i} \asymp a_i,\ \tilde{a}_{2i} \asymp \tilde{a}_i \tag{3.5}$$

or

$$b_{2j} \asymp b_j,\ \tilde{b}_{2j} \asymp \tilde{b}_j. \tag{3.6}$$

Then $X \overset{qd}{\simeq} Y$ (and all the more $X \simeq Y$).

Proof. Indeed, both of the conditions (3.7), (3.8) supply $X \overset{qd}{\simeq} X^2$ and $Y \overset{qd}{\simeq} Y^2$, hence we can apply Theorem 3.8. □

Theorem 3.11. *Let $X = s'\hat{\otimes}E_\infty(\tilde{a})$ be nuclear. Then the following statements are equivalent:*

(i) $X \overset{(qd)}{\simeq} Y$

(ii) $X \simeq Y$

(iii) $\exists\, A\ \forall\, \gamma > 0\ \exists\, \tau_0$ *such that*

$$m_a(\tau, t) \leq (\exp \gamma t) m_{\tilde{a}}(\frac{\tau}{A}, At), \quad \tau_0 \leq \tau \leq t \tag{3.7}$$

$$m_{\tilde{a}}(\tau, t) \leq (\exp \gamma t) m_a(\frac{\tau}{A}, At), \quad \tau_0 \leq \tau \leq t \tag{3.8}$$

(iv) $\forall\, A\ \exists\, B\ \forall\, \gamma\ \exists\, \tau_0$ *such that*

$$m_a(\frac{t}{A}, At) \leq (\exp\gamma t) m_{\tilde{a}}(\frac{t}{B}, Bt), t \geq \tau_0 \tag{3.9}$$

$$m_{\tilde{a}}(\frac{t}{A}, At) \leq (\exp \gamma t) m_a(\frac{t}{B}, Bt), t \geq \tau_0 \tag{3.10}$$

and $\exists\, E > 1$ *such that*

$$m_a(t) \leq (\exp Et) m_{\tilde{a}}(Et)$$

$$m_{\tilde{a}}(t) \leq (\exp Et) m_a(Et) \quad \textit{for} \quad t \geq 1$$

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